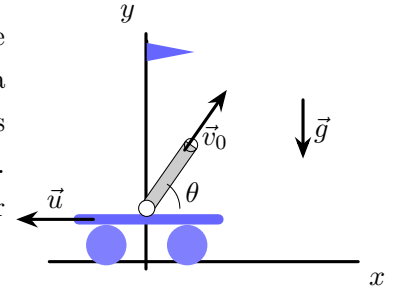


1. A cannon is on a train car traveling with velocity $u = 2$ m/s to the left. The cannon fires at an angle θ with the horizontal as it passes a flag post fixed to the ground (see the figure). The cannonball leaves the cannon with speed $v_0 = 25$ m/s (with respect to the cannon). (Gravitational acceleration is $g = 10$ m/s², the height of the train car and the cannon is negligible, and $\cos \theta = 4/5$.)



- [7 Pts.] (a) What is the velocity of the cannonball with respect to the ground?

$$\text{With respect to the train: } \vec{v}_{CT} = 25 \times \frac{4}{5} \hat{i} + 25 \times \frac{3}{5} \hat{j} = 20\hat{i} + 15\hat{j} \text{ m/s}$$

$$\text{With respect to the ground: } \vec{v}_{CG} = \vec{v}_{CT} - 2\hat{i} = 18\hat{i} + 15\hat{j} \text{ m/s}$$

- [9 Pts.] (b) How far away from the post does the cannonball hit the ground?

$$\text{Time of flight: } y = v_y t - \frac{1}{2} g t^2 \quad \text{with } y = 0 \quad \Rightarrow \quad t_f = \frac{2v_y}{g} = \frac{30}{10} = 3 \text{ s}$$

$$\text{Then, } x_f = v_x t_f = 18 \times 3 = 54 \text{ m}$$

- [9 Pts.] (c) With $|\vec{v}_0|$ fixed, the angle of the cannon can be changed to change the range. For which value of θ is the distance from the post to the hit point on the ground is maximum? (You can express your answer in terms of inverse trigonometric functions.)

$$\text{For any } \theta: \vec{v}_{CG} = (|\vec{v}_0| \cos \theta - u) \hat{i} + (|\vec{v}_0| \sin \theta) \hat{j}$$

$$\text{for max. range } \frac{d}{d\theta} \left[(v_0 \cos \theta - u) \frac{2v_0 \sin \theta}{g} \right] = 0$$

$$v_0 (\cos^2 \theta - \sin^2 \theta) - u \cos \theta = 0$$

$$v_0 \cos 2\theta - u \cos \theta = 0 \quad \Rightarrow \quad 2v_0 \cos^2 \theta - u \cos \theta - v_0 = 0$$

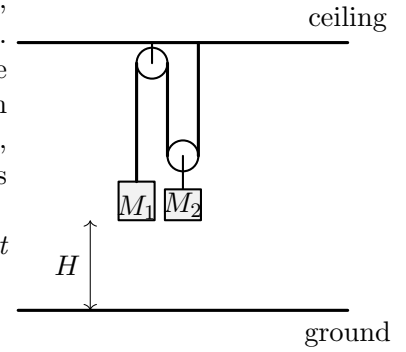
$$\cos \theta = \frac{u \pm \sqrt{u^2 + 8v_0^2}}{4v_0} \quad \Rightarrow \quad \theta = \arccos \left[\frac{1}{2} \left(\frac{1}{25} \pm \sqrt{\left(\frac{1}{25} \right)^2 + 2} \right) \right]$$

$$\text{Another form of the solution is } \sin^2 \theta = \frac{4v_0^2 - u^2 \mp u \sqrt{u^2 + 8v_0^2}}{8v_0^2}$$

2. Two masses M_1 and M_2 are connected by a single ideal rope (massless, inextensible) that runs vertically except where it passes over pulleys. The rope is arranged as follows: one end holds mass M_1 , the rope goes up over a *fixed* pulley attached to the ceiling, then goes down and passes under a *movable* massless pulley that supports mass M_2 , and finally goes up and is attached to the ceiling. Assume all pulleys are frictionless and massless.

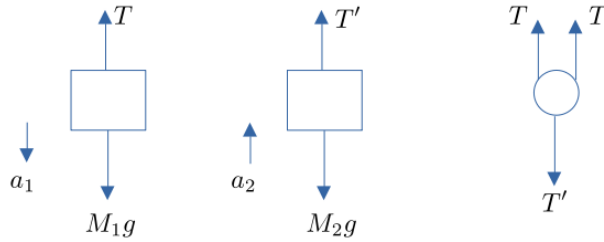
The system is released from rest. Initially, *both masses are at height H above the ground.*

For the numerical values $M_1 = 1.0$ kg, $M_2 = 1.0$ kg, $g = 10$ m/s², $H = 5.0$ m, answer the following:



[5 Pts.]

- (a) Draw free body diagram for the masses, and the moving pulley.



[15 Pts.]

- (b) Calculate the accelerations of the masses. Which mass reaches the ground first ?

$$M_1 a_1 = M_1 g - T$$

$$M_2 a_2 = 2T - M_2 g$$

If M_1 moves down with Δ_1 , and M_2 moves up with Δ_2 the change in the length of the rope is $\Delta_1 - 2\Delta_2 = 0$

$$\Rightarrow a_1 = 2a_2$$

$$\Rightarrow M_1(2a_2) = M_1 g - T \quad \text{and} \quad M_2 a_2 = 2T - M_2 g$$

$$a_2 = \frac{(2M_1 - M_2)g}{(4M_1 + M_2)} = \frac{(2 - 1)10}{5} = 2 \text{ m/s}^2 \quad \text{moves up, and} \quad a_1 = 4 \text{ m/s}^2 \quad \text{moves down}$$

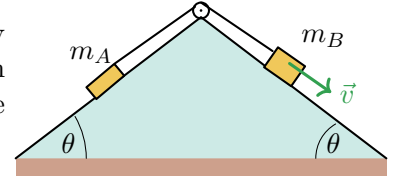
M_1 moves downward, it reaches the ground first.

[5 Pts.]

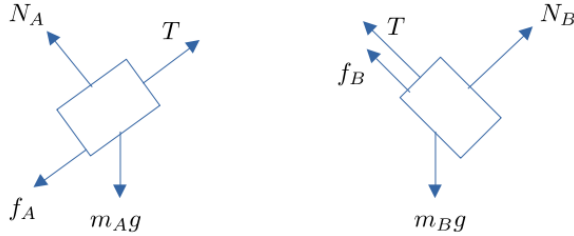
- (c) How long does it take (from release) for that mass to reach the ground?

$$H = \frac{1}{2} a_1 t_f^2 \quad \Rightarrow \quad t_f = \sqrt{\frac{2H}{a_1}} = \sqrt{\frac{5}{2}} \text{ s}$$

3. Two masses m_A and m_B are on inclines and are connected together by a string as shown in figure. The coefficient of kinetic friction between each mass and its incline is μ , (take $\mu_s = \mu_k$). Ignore masses of the cord and the pulley (assume frictionless pulleys).



- [5 Pts.] (a) Draw free body diagram for the masses.



- [10 Pts.] (b) If m_A moves up the incline, and m_B moves down, determine their acceleration.

$$N_A = m_A g \cos \theta \quad \text{and} \quad N_B = m_B g \cos \theta$$

$$T - m_A g \sin \theta - f_A = m_A a$$

$$m_B g \sin \theta - T - f_B = m_B a$$

Adding them up yields

$$(m_B - m_A)g \sin \theta - (m_A + m_B)\mu g \cos \theta = (m_A + m_B)a$$

$$\Rightarrow a = \frac{(m_B - m_A)}{(m_A + m_B)}g \sin \theta - \mu g \cos \theta$$

- [10 Pts.] (c) For which angles θ the blocks will start to move if released from the rest?

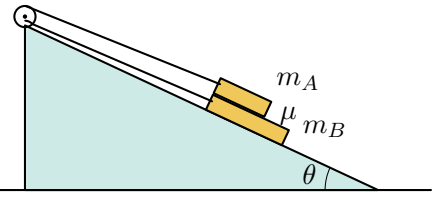
The above formula shows $a > 0$ if

$$\tan \theta \geq \mu \frac{(m_B + m_A)}{|m_B - m_A|}$$

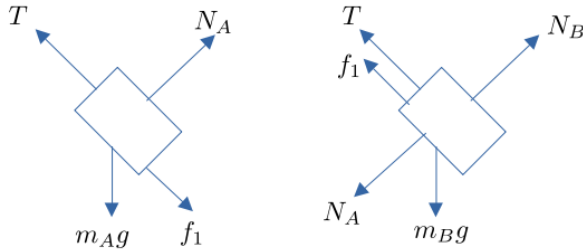
or

$$\theta \geq \arctan \left[\mu \frac{(m_B + m_A)}{|m_B - m_A|} \right]$$

4. A block B of mass m_B rests on a static, rigid incline. The incline makes an angle θ with the horizontal surface. The contact between block B and the incline is frictionless. A second block A of mass m_A rests on top of B . The coefficient of friction between A and B is μ (take $\mu_s = \mu_k = \mu$). Blocks are connected by a light inextensible string that passes over a fixed, massless, frictionless pulley mounted at the top of the incline. Both string segments are parallel to the incline.



- [5 Pts.] (a) Draw the free-body diagrams of blocks A and B .



- [10 Pts.] (b) Determine for which θ the upper block A can remain at rest relative to B (static equilibrium).

$$\begin{aligned}
 N_A &= m_A g \cos \theta \quad \text{and} \quad N_B = (m_A + m_B) g \cos \theta \\
 T - m_A g \sin \theta &= f_1 \quad \text{and} \quad T + f_1 = m_B g \sin \theta \\
 \Rightarrow f_1 &= \frac{m_B - m_A}{2} g \sin \theta \\
 |f_1| < \mu N_A &\Rightarrow \frac{|m_B - m_A|}{2} g \sin \theta < \mu m_A g \cos \theta \\
 \tan \theta &< \frac{2\mu m_A}{|m_B - m_A|}
 \end{aligned}$$

- [10 Pts.] (c) If the friction coefficient smaller than its minimum value μ_{min} required to keep A from slipping relative to B , find the relative acceleration of m_A with respect to m_B .

$$\begin{aligned}
 T - m_A g \sin \theta - \mu m_A g \cos \theta &= m_A a \\
 m_B g \sin \theta - T - \mu m_A g \cos \theta &= m_B a \\
 \Rightarrow a &= \frac{1}{(m_A + m_B)} [(m_B - m_A) g \sin \theta - 2\mu m_A g \cos \theta]
 \end{aligned}$$